ARITHMETIC PROGRESSIONS — EXERCISE

1) The 4^{th} term of an A.P. is 19 and the 11^{th} term is 54. Find the first term a, the common difference d, and the sum of the first 20 terms S_{20}.

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2) The following sequence is an A.P. 91, 88, 85, 82, ...
Find the sum of terms 11 to 20 inclusive

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3) The first term of an A.P. is 65, and the common difference is -2.5. The r^{th} term $= 0$, find the value of r.
If S_n is the sum of the first n terms, find the maximum value of S_n.

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4) The 3rd term of an A.P. is 7, and the sum of the first 10 terms is 120. Find the first term a, and the common difference d.

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5) In an A.P. the first term $a = 12$, and the common difference $d = 4$. The sum of the first n terms $S_n = 600$. Find n.

ARITHMETIC PROGRESSIONS — EXERCISE

1) 4^{th} term $\quad a + 3d = 19 \quad (1)$

11^{th} term $\quad a + 10d = 54 \quad (2)$

$(2) - (1) \quad 7d = 35$

$\Rightarrow d = \frac{35}{7}$

$\Rightarrow \underline{d = 5}$

Subst for d in (1)

$a + 3(5) = 19$

$a + 15 = 19$

$a = 19 - 15$

$\underline{a = 4}$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = \frac{20}{2}(8 + 5 \times 19)$

$S_{20} = 10 \times 103$

$\underline{S_{20} = 1030}$

Answers: $a = 4$, $d = 5$, $S_{20} = 1030$

2) 91, 88, 85, 82, ... $a = 91$, $d = -3$

Sum of terms 11 to 20 inclusive is given by $S_{20} - S_{10}$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_{20} = \frac{20}{2}\left(2\times 91 + 19\times(-3)\right)$$

$$= 10\left(182 - 57\right)$$

$$= 10 \times 125$$

$$S_{20} = 1250$$

$$S_{10} = \frac{10}{2}\left(2\times 91 + 9\times(-3)\right)$$

$$= 5\left(182 - 27\right)$$

$$= 5 \times 155$$

$$S_{10} = 775$$

$$S_{20} - S_{10} = 1250 - 775$$

$$S_{20} - S_{10} = 475$$

Sum of terms 11 to 20 inclusive = 475

3) $a = 65$, $d = -2.5$

r^{th} term $= 0$

Since n^{th} term $= a + (n-1)d$

r^{th} term $= a + (r-1)d$

$\therefore \quad a + (r-1)d = 0$

$65 - 2.5(r-1) = 0$

$130 - 5(r-1) = 0$

$130 - 5r + 5 = 0$

$135 = 5r$

$r = \frac{135}{5}$

$\underline{r = 27}$

S_n will increase while n increases until terms become negative. The last non-negative term will be term 27 which is equal to 0 from first part of question

So find S_{26} or S_{27} to give maximum S_n

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$S_{26} = \frac{26}{2}\left(130 - 2.5(25)\right)$

$S_{26} = 13 \times 67.5$

$S_{26} = 877.5$

Check with S_{27}

$S_{27} = \frac{27}{2}\left(130 - 2.5(26)\right)$

$S_{27} = 13.5 \times 65$

$S_{27} = 877.5$

$\underline{\text{Maximum value of } S_n = 877.5}$

4) 3rd term $a + 2d = 7$

$S_{10} = 120$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = \frac{10}{2}(2a + 9d) = 120$

$\Rightarrow \quad 5(2a + 9d) = 120$

$2a + 9d = \frac{120}{5} = 24$

$\therefore \quad a + 2d = 7 \quad (1)$

$2a + 9d = 24 \quad (2)$

(1) x 2 $\quad 2a + 4d = 14 \quad (3)$

(2) − (3) $\quad 5d = 10$

$\Rightarrow \underline{d = 2}$

Subst for d in (1)

$a + 2(2) = 7$

$a + 4 = 7$

$a = 7 - 4$

$\underline{a = 3}$

Answer: $a = 3, \; d = 2$

5) $a = 12, \quad d = 4, \quad S_n = 600$ Find n

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right) = 600$$

$$\Rightarrow \frac{n}{2}\left(24 + 4(n-1)\right) = 600$$

$$n\left(24 + 4n - 4\right) = 1200$$

$$4n^2 + 20n - 1200 = 0$$

$$n^2 + 5n - 300 = 0$$

$$(n-15)(n+20) = 0$$

$$\Rightarrow n = 15 \text{ or } n = \cancel{-20}$$

not relevant to question

Answer $n = 15$