LINEAR AND QUADRATIC INEQUALITIES TRANSCRIPT

Consider operations on the inequality:

$$4 < 6$$

Add 1 to both sides

$$5 < 7 \quad \text{still true}$$

Subtract 1 from both sides

$$3 < 5 \quad \text{still true}$$

Multiply each side by 2

$$8 < 12 \quad \text{still true}$$

Divide each side by 2

$$2 < 3 \quad \text{still true}$$

However,

Multiply each side by -2

$$-8 > -12$$

inequality has been reversed

Divide each side by -2

$$-2 > -3$$

again the inequality has been reversed

So when we are solving linear inequalities we can treat them very much like linear equations. However, we need to bear in mind that multiplying or dividing an inequality by a negative number causes the direction of the inequality to be reversed.

Example 1

Solve $2x + 3 < 11$

$$2x < 11 - 3$$

$$2x < 8$$

$$x < \frac{8}{2}$$

$$x < 4$$

Example 2

Solve $3x + 9 \geqslant 7x + 33$

$$3x - 7x \geqslant +33 - 9$$

$$-4x \geqslant 24$$

$$x \leqslant \frac{24}{-4}$$

$$x \leqslant -6$$

LINEAR AND QUADRATIC INEQUALITIES TRANSCRIPT

Example 3

Solve $5(x+2) \geq 3x - 9$

$5x + 10 \geq 3x - 9$

$5x - 3x \geq -9 - 10$

$2x \geq -19$

$x \geq -\frac{19}{2}$

$x \geq -\frac{19}{2}$

Quadratic Inequalities

Example 4

Solve $x^2 - 6x + 8 < 0$

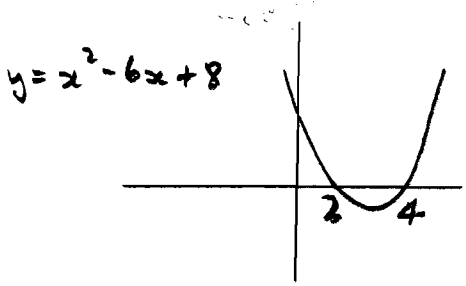
First factorise left hand side

$(x-2)(x-4) < 0$

Now sketch graph of

$y = x^2 - 6x + 8$

using the above factors to identify where graph cuts x-axis



Note that $y = 0$ when $x = 2$ and when $x = 4$

Solving the inequality is equivalent to identifying where $y < 0$. From the graph we can see that this is between $x = 2$ and $x = 4$. The solution should be written as

$$2 < x < 4$$

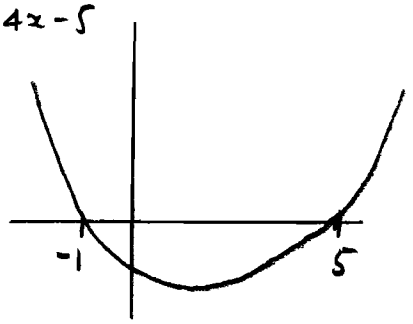
since both inequalities must be true at the same time.

Example 5

Solve $x^2 - 4x - 5 \geq 0$

$(x-5)(x+1) \geqslant 0$

$y = x^2 - 4x - 5$



This time we require $y \geqslant 0$ which is true for:

Either $x \leqslant -1$

or $x \geqslant 5$

These relationships should be

stated either/or because clearly they cannot be true at the same time

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Example 6

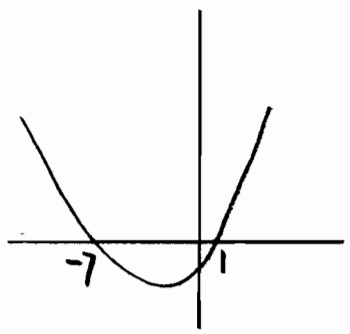
Solve $7 - 6x - x^2 > 0$

Move all terms to the other side so that x^2 term is positive. Then proceed as before

$$0 > x^2 + 6x - 7$$

$$0 > (x+7)(x-1)$$

$y = x^2 + 6x - 7$



We require $y < 0$ so

$$-7 < x < 1$$

Example 7

Solve $3x^2 - 2x - 8 > 0$

Factorise

$3x - 8 = -24$

We require factors of -24 which add together to give -2

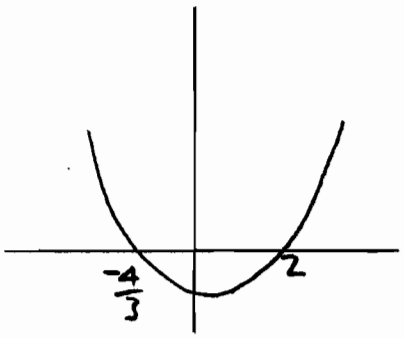
so -6 and $+4$ giving

$$3x^2 - 6x + 4x - 8 > 0$$

$$3x(x-2) + 4(x-2) > 0$$

$$(3x+4)(x-2) > 0$$

Graph of $y = 3x^2 - 2x - 8$



Note

$y = 0$ when $x - 2 = 0 \Rightarrow x = 2$

$y = 0$ when $3x + 4 = 0 \Rightarrow x = -\frac{4}{3}$

We require $y > 0$ so solution is

Either $x > 2$

or $x < -\frac{4}{3}$
