FACTOR AND REMAINDER THEOREMS — EXERCISE

1) Find the remainder when $f(x) = x^3 + 2x^2 + 3x + 4$ is divided by $(x-3)$

2) Find the remainder when $f(x) = x^3 + 2x^2 + 3x + 4$ is divided by $(x+4)$

3) When $f(x) = x^3 - x^2 + kx + 2$ is divided by $(x-5)$ the remainder is 127. Find k

4) Let $f(x) = x^3 - 6x^2 + 3x + 10$

Use the factor theorem to fully factorise $f(x)$ and hence sketch the graph of $y = f(x)$

5) Use the factor theorem to find a root of the equation

$$x^3 + 2x^2 - 6x - 4 = 0$$

Then use algebraic long division to help find the other two roots, giving those roots in surd form

FACTOR AND REMAINDER THEOREMS EXERCISE

1) $f(x) = x^3 + 2x^2 + 3x + 4$

Remainder when $f(x)$ is divided by $(x-3)$ is given by $f(3)$

$f(3) = 3^3 + 2(3)^2 + 3(3) + 4$

$= 27 + 18 + 9 + 4$

$= 58$

2) $f(x) = x^3 + 2x^2 + 3x + 4$

Remainder when $f(x)$ is divided by $(x+4)$ is given by $f(-4)$

$f(-4) = (-4)^3 + 2(-4)^2 + 3(-4) + 4$

$= -64 + 32 - 12 + 4$

$= -40$

3) $f(x) = x^3 - x^2 + kx + 2$

when divided by $(x-5)$ remainder is 127

$\therefore \quad f(5) = 127$

$5^3 - 5^2 + 5k + 2 = 127$

$125 - 25 + 5k + 2 = 127$

$5k = 127 - 125 + 25 - 2$

$5k = 25$

$k = \frac{25}{5}$

$k = 5$.

4) $f(x) = x^3 - 6x^2 + 3x + 10$

Could try $\pm 1, \pm 2, \pm 5, \pm 10$

$f(1) = 1^3 - 6(1)^2 + 3(1) + 10$

$= 1 - 6 + 3 + 10$

$= 8$ ✗

$f(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10$

$= -1 - 6 - 3 + 10$

$= 0$ ✓

$f(2) = 2^3 - 6(2)^2 + 3(2) + 10$

$= 8 - 24 + 6 + 10$

$= 0$ ✓

$f(x) = (x-2)(x+1)(x \quad)$

To give constant term of +10 the missing number must be -5 but check $f(5)$ to confirm this

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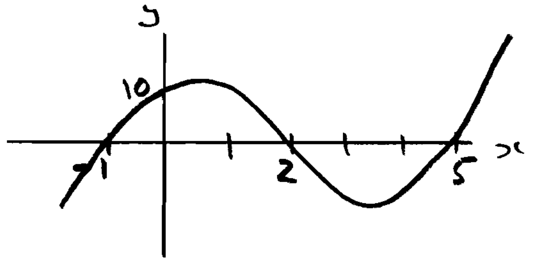
4 cont)

$f(5) = 5^3 - 6(5)^2 + 3(5) + 10$

$= 125 - 150 + 15 + 10$

$= 0$ ✓

So $f(x) = (x-2)(x+1)(x-5)$



5) Let $f(x) = x^3 + 2x^2 - 6x - 4$

Solve $f(x) = 0$

Could try $\pm 1, \pm 2, \pm 4$

$f(1) = 1^3 + 2(1)^2 - 6(1) - 4$

$= 1 + 2 - 6 - 4$

$= -7$ ✗

$f(-1) = (-1)^3 + 2(-1)^2 - 6(-1) - 4$

$= -1 + 2 + 6 - 4$

$= 3$ ✗

$f(2) = 2^3 + 2(2)^2 - 6(2) - 4$

$= 8 + 8 - 12 - 4$

$= 0$ ✓

$\therefore (x-2)$ a factor

$$\begin{array}{r|l} & x^2 + 4x + 2 \\ \hline x-2 & x^3 + 2x^2 - 6x - 4 \\ & x^3 - 2x^2 \\ \hline & +4x^2 - 6x \\ & +4x^2 - 8x \\ \hline & +2x - 4 \\ & +2x - 4 \\ \hline \end{array}$$

$f(x) = (x-2)(x^2 + 4x + 2)$

For $f(x) = 0$

Either $(x-2) = 0 \Rightarrow x = 2$

or $(x^2 + 4x + 2) = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 - 8}}{2}$$

$$x = \frac{-4 \pm \sqrt{8}}{2}$$

$$x = \frac{-4 \pm \sqrt{4 \times 2}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{2}}{2}$$

$$x = -2 \pm \sqrt{2}$$

Solution

$x = 2,\ x = -2 + \sqrt{2},\ x = 2 - \sqrt{2}$