INTEGRATION OF POLYNOMIAL FUNCTIONS — EXERCISE

1. Find the following indefinite integrals

a) $\int (x^3 - x^2)\,dx$

b) $\int (6x + 3)\,dx$

c) $\int (5x^2 - 4x)\,dx$

d) $\int (4x^3 - 3x^2 + 2x - 1)\,dx$

e) $\int \left(\frac{1}{2}x^3 + \frac{1}{3}x\right)\,dx$

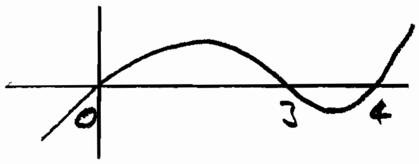
2. The gradient function of a curve is $\frac{dy}{dx} = 5x - 2$ and the curve passes through $(6, 3)$. Find the equation of the curve.

3. The gradient function of a curve is $\frac{dy}{dx} = 4x^3 - x$ and the curve passes through $(2, -5)$. Find the equation of the curve.

4. Find the area between the curve $y = x^2 + x$, the x-axis and the lines $x = 1$ and $x = 3$

5. $y = x(x-3)(x-4)$

$y = x^3 - 7x^2 + 12x$



Find the area between the curve and the x-axis from $x = 0$ to $x = 3$

6. Evaluate the definite integrals

a) $\int_1^3 (x^2 - 1)\,dx$

b) $\int_0^2 (x^2 - x + 1)\,dx$

c) $\int_{-1}^2 (x^4 + x^2)\,dx$

d) $\int_0^3 \left(6x^2 + \frac{1}{3}x\right)\,dx$

e) $\int_{-3}^{-1} (6x - 5)\,dx$

INTEGRATION OF POLYNOMIAL FUNCTIONS — EXERCISE

1)

a) $\int (x^3 - x^2)\,dx$

$= \frac{x^4}{4} - \frac{x^3}{3} + c$

b) $\int (6x + 3)\,dx$

$= \frac{6x^2}{2} + 3x + c$

$= 3x^2 + 3x + c$

c) $\int (5x^2 - 4x)\,dx$

$= \frac{5x^3}{3} - \frac{4x^2}{2} + c$

$= \frac{5x^3}{3} - 2x^2 + c$

d) $\int (4x^3 - 3x^2 + 2x - 1)\,dx$

$= \frac{4x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2} - x + c$

$= x^4 - x^3 + x^2 - x + c$

e) $\int \left(\frac{1}{2}x^3 + \frac{1}{3}x\right)dx$

$= \frac{1}{2}\cdot\frac{x^4}{4} + \frac{1}{3}\cdot\frac{x^2}{2} + c$

$= \frac{x^4}{8} + \frac{x^2}{6} + c$

2. $\frac{dy}{dx} = 5x - 2$

$\Rightarrow y = \frac{5x^2}{2} - 2x + c$

Subst $(6, 3)$

$3 = \frac{5 \times 6^2}{2} - 2(6) + c$

$3 = 90 - 12 + c$

$3 - 90 + 12 = c$

$-75 = c$

$\therefore y = \frac{5x^2}{2} - 2x - 75$

3. $\frac{dy}{dx} = 4x^3 - x$

$\Rightarrow y = \frac{4x^4}{4} - \frac{x^2}{2} + c$

$y = x^4 - \frac{x^2}{2} + c$

Subst $(2, -5)$

$-5 = 2^4 - \frac{2^2}{2} + c$

$-5 = 16 - 2 + c$

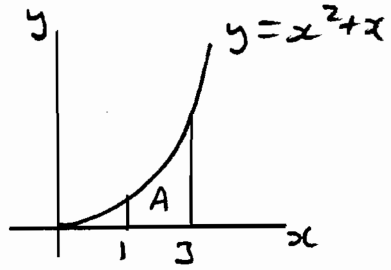
$-5 - 16 + 2 = c$

$-19 = c$

$\therefore y = x^4 - \frac{x^2}{2} - 19$

INTEGRATION OF POLYNOMIAL FUNCTIONS EXERCISE

4.



$$A = \int_1^3 (x^2+x)\,dx$$

$$= \left[\frac{x^3}{3}+\frac{x^2}{2}\right]_1^3$$

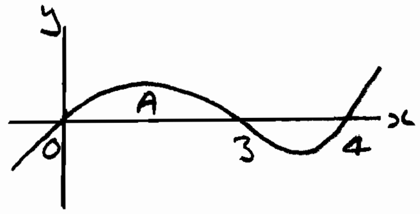
$$= \left(\frac{3^3}{3}+\frac{3^2}{2}\right)-\left(\frac{1^3}{3}+\frac{1^2}{2}\right)$$

$$= \left(9+\frac{9}{2}\right)-\left(\frac{1}{3}+\frac{1}{2}\right)$$

$$= 13\tfrac{1}{2} - \tfrac{5}{6}$$

$$= 12\tfrac{2}{3} \text{ units}^2$$

5.



$$A = \int_0^3 (x^3-7x^2+12x)\,dx$$

$$= \left[\frac{x^4}{4}-\frac{7x^3}{3}+\frac{12x^2}{2}\right]_0^3$$

$$= \left[\frac{x^4}{4}-\frac{7x^3}{3}+6x^2\right]_0^3$$

$$= \left(\frac{3^4}{4}-\frac{7(3)^3}{3}+6(3)^2\right)-\left(0-0+0\right)$$

$$= \frac{81}{4} - 63 + 54$$

$$= 20\tfrac{1}{4} - 9$$

$$= 11\tfrac{1}{4} \text{ units}^2$$

6 a)

$$\int_1^3 (x^2-1)\,dx$$

$$= \left[\frac{x^3}{3}-x\right]_1^3$$

$$= \left(\frac{3^3}{3}-3\right)-\left(\frac{1^3}{3}-1\right)$$

$$= 6 - -\frac{2}{3}$$

$$= 6\tfrac{2}{3}$$

6 b)

$$\int_0^2 (x^2-x+1)\,dx$$

$$= \left[\frac{x^3}{3}-\frac{x^2}{2}+x\right]_0^2$$

$$= \left(\frac{2^3}{3}-\frac{2^2}{2}+2\right)-\left(0-0+0\right)$$

$$= \frac{8}{3} - 2 + 2$$

$$= \frac{8}{3} = 2\tfrac{2}{3}$$

INTEGRATION OF POLYNOMIAL FUNCTIONS

EXERCISE

6c) $$\int_{-1}^{2} (x^4 + x^2)\, dx$$

$$= \left[\frac{x^5}{5} + \frac{x^3}{3} \right]_{-1}^{2}$$

$$= \left(\frac{2^5}{5} + \frac{2^3}{3} \right) - \left(\frac{(-1)^5}{5} + \frac{(-1)^3}{3} \right)$$

$$= \left(\frac{32}{5} + \frac{8}{3} \right) - \left(-\frac{1}{5} - \frac{1}{3} \right)$$

$$= \frac{32}{5} + \frac{8}{3} + \frac{1}{5} + \frac{1}{3}$$

$$= \frac{33}{5} + \frac{9}{3}$$

$$= 9\frac{3}{5}$$

6e) $$\int_{-3}^{-1} (6x - 5)\, dx$$

$$= \left[\frac{6x^2}{2} - 5x \right]_{-3}^{-1}$$

$$= \left[3x^2 - 5x \right]_{-3}^{-1}$$

$$= \left(3(-1)^2 - 5(-1) \right) - \left(3(-3)^2 - 5(-3) \right)$$

$$= (3 + 5) - (27 + 15)$$

$$= 8 - 42$$

$$= -34$$

6d) $$\int_{0}^{3} \left(6x^2 + \frac{1}{3}x \right) dx$$

$$= \left[\frac{6x^3}{3} + \frac{1}{3} \cdot \frac{x^2}{2} \right]_{0}^{3}$$

$$= \left[2x^3 + \frac{x^2}{6} \right]_{0}^{3}$$

$$= \left(2(3)^3 + \frac{3^2}{6} \right) - (0 + 0)$$

$$= 54 + \frac{9}{6}$$

$$= 55\frac{1}{2}$$