DIFFERENTIATION OF POLYNOMIALS (1)   EXERCISE

1. Find $\dfrac{dy}{dx}$ for the following functions

a. $y = 3x$ f. $y = x^5 - x^4 + x^3$

b. $y = 4x^7$ g. $y = \frac{1}{2}x^3 + \frac{1}{3}x^2$

c. $y = -3x^3$ h. $y = x^2 - x + 1$

d. $y = -4$ i. $y = x^{10} - x^{20}$

e. $y = \frac{1}{2}x^2$ j. $y = 3x^7 + 5x^4 - 2x$

2. Consider the curve $y = x^3 - 6x^2 + 3$

i) Find $\dfrac{dy}{dx}$

ii) Find the eqn of the tangent where $x = 1$

iii) Find the eqn of the normal where $x = 2$

3. Consider the curve $y = \frac{1}{3}x^3 - \frac{5}{2}x^2$

Find the coords of the points on this curve where the

gradient $= 6$

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DIFFERENTIATION OF POLYNOMIALS (1)   EXERCISE

1.

a.
$$y = 3x$$
$$\frac{dy}{dx} = 3$$

b.
$$y = 4x^7$$
$$\frac{dy}{dx} = 28x^6$$

c.
$$y = -3x^3$$
$$\frac{dy}{dx} = -9x^2$$

d.
$$y = -4$$
$$\frac{dy}{dx} = 0$$

e.
$$y = \frac{1}{2}x^2$$
$$\frac{dy}{dx} = x$$

f.
$$y = x^5 - x^4 + x^3$$
$$\frac{dy}{dx} = 5x^4 - 4x^3 + 3x^2$$

g.
$$y = \frac{1}{2}x^3 + \frac{1}{3}x^2$$
$$\frac{dy}{dx} = \frac{3}{2}x^2 + \frac{2}{3}x$$

h.
$$y = x^2 - x + 1$$
$$\frac{dy}{dx} = 2x - 1$$

i.
$$y = x^{10} - x^{20}$$
$$\frac{dy}{dx} = 10x^9 - 20x^{19}$$

j.
$$y = 3x^7 + 5x^4 - 2x$$
$$\frac{dy}{dx} = 21x^6 + 20x^3 - 2$$

2.
$$y = x^3 - 6x^2 + 3$$

i)
$$\frac{dy}{dx} = 3x^2 - 12x$$

ii) When $x = 1$, $y = 1^3 - 6(1)^2 + 3$
$$y = 1 - 6 + 3 = -2$$

Point on curve is $(1, -2)$

When $x = 1$, $\frac{dy}{dx} = 3(1)^2 - 12(1)$
$$= 3 - 12 = -9$$

so gradient $= -9$

tgt $\quad y - y_1 = m(x - x_1)$
$$y - -2 = -9(x - 1)$$
$$y + 2 = -9x + 9$$
$$y = -9x + 7$$

2iii) When $x = 2$

$$y = 2^3 - 6(2)^2 + 3$$

$$y = 8 - 24 + 3 = -13$$

Point is $(2, -13)$

When $x = 2$, $\dfrac{dy}{dx} = 3(2)^2 - 12(2)$

$$= 12 - 24 = -12$$

$\therefore$ grad of normal $= +\dfrac{1}{12}$

Using $y - y_1 = m(x - x_1)$

$$y - -13 = \frac{1}{12}(x - 2)$$

$$y + 13 = \frac{1}{12}x - \frac{2}{12}$$

$$y = \frac{1}{12}x - \frac{2}{12} - \frac{156}{12}$$

Normal

$$y = \frac{1}{12}x - \frac{158}{12}$$

or $x - 12y - 158 = 0$

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3. $y = \frac{1}{3}x^3 - \frac{5}{2}x^2$

$\Rightarrow \dfrac{dy}{dx} = x^2 - 5x$

When gradient $= 6$

$$x^2 - 5x = 6$$

$$x^2 - 5x - 6 = 0$$

$$(x + 1)(x - 6) = 0$$

$\Rightarrow x = -1$ or $x = 6$

When $x = -1$

$$y = \frac{1}{3}(-1)^3 - \frac{5}{2}(-1)^2$$

$$y = -\frac{1}{3} - \frac{5}{2} = -\frac{2}{6} - \frac{15}{6}$$

$$y = -\frac{17}{6}$$

Point is $\left(-1, -\frac{17}{6}\right)$

When $x = 6$

$$y = \frac{1}{3}6^3 - \frac{5}{2}(6)^2$$

$$= \frac{216}{3} - \frac{5 \times 36}{2}$$

$$= 72 - 90 = -18$$

Point is $(6, -18)$

Gradient $= 6$ at points

$$\left(-1, -\frac{17}{6}\right) \text{ and } \left(6, -18\right)$$