

# INTEGRATION BY SUBSTITUTION

①

## Exercise

1.  $\int x(2x-1)^3 dx$

2.  $\int \frac{2x}{x+3} dx$

3.  $\int 2x\sqrt{4x+1} dx$

4.  $\int 2x\sqrt{3x^2+1} dx$

5.  $\int 8\cos x \sin^4 x dx$

6.  $\int_5^{10} \frac{x}{\sqrt{x-1}} dx$

7.  $\int_0^1 5x^2 e^{x^3} dx$

INTEGRATION BY SUBSTITUTION

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1.  $\int x(2x-1)^3 dx$

Let  $u = 2x - 1$

$\Rightarrow \frac{du}{dx} = 2$

$\Rightarrow du = 2dx$

$\Rightarrow \frac{1}{2} du = dx$

Also  $u + 1 = 2x$

$\Rightarrow \frac{u+1}{2} = x$

$= \int \left(\frac{u+1}{2}\right) u^3 \frac{1}{2} du$

$= \frac{1}{4} \int (u+1)u^3 du$

$= \frac{1}{4} \int (u^4 + u^3) du$

$= \frac{1}{4} \left( \frac{u^5}{5} + \frac{u^4}{4} \right) + C$

$= \frac{u^5}{20} + \frac{u^4}{16} + C$

$= \frac{(2x-1)^5}{20} + \frac{(2x-1)^4}{16} + C$

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INTEGRATION BY SUBSTITUTION

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2.  $\int \frac{2x}{x+3} dx$

Let  $u = x+3$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Also  $x = u-3$

$= \int \frac{2(u-3)}{u} du$

$= \int \frac{2u-6}{u} du$

$= \int \left( \frac{2u}{u} - \frac{6}{u} \right) du$

$= \int \left( 2 - \frac{6}{u} \right) du$

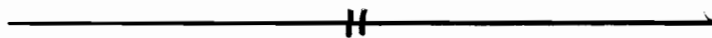
$= 2u - 6 \ln u + c$

$= 2(x+3) - 6 \ln|x+3| + c$

$= 2x + 6 - 6 \ln|x+3| + c$

$= 2x - 6 \ln|x+3| + c$

since constant  $c$  can absorb constant  $+6$



INTEGRATION BY SUBSTITUTION

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3.  $\int 2x \sqrt{4x+1} dx$

Let  $u = 4x+1$

$\Rightarrow \frac{du}{dx} = 4$

$\Rightarrow du = 4 dx$

$\Rightarrow \frac{1}{2} du = 2 dx$

Also  $u-1 = 4x$

$\Rightarrow \frac{u-1}{4} = x$

$= \int \left(\frac{u-1}{4}\right) u^{\frac{1}{2}} \frac{1}{2} du$

$= \frac{1}{8} \int (u-1) u^{\frac{1}{2}} du$

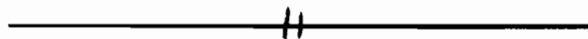
$= \frac{1}{8} \int (u^{3/2} - u^{1/2}) du$

$= \frac{1}{8} \left( \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} \right) + C$

$= \frac{1}{8} \left( \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right) + C$

$= \frac{1}{20} u^{5/2} - \frac{1}{12} u^{3/2} + C$

$= \frac{1}{20} (4x+1)^{5/2} - \frac{1}{12} (4x+1)^{3/2} + C$



INTEGRATION BY SUBSTITUTION

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4.  $\int 2x \sqrt{3x^2+1} dx$

Let  $u = 3x^2+1$

$\Rightarrow \frac{du}{dx} = 6x$

$\Rightarrow du = 6x dx$

$\Rightarrow \frac{1}{3} du = 2x dx$

$= \int u^{\frac{1}{2}} \frac{1}{3} du$

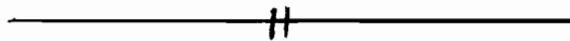
$= \frac{1}{3} \int u^{\frac{1}{2}} du$

$= \frac{1}{3} \left( \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$

$= \frac{1}{3} \left( \frac{2}{3} u^{\frac{3}{2}} \right) + C$

$= \frac{2}{9} u^{\frac{3}{2}} + C$

$= \frac{2}{9} (3x^2+1)^{\frac{3}{2}} + C$



## INTEGRATION BY SUBSTITUTION

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$$5. \int 8 \cos x \sin^4 x \, dx$$

$$\text{Let } u = \sin x$$

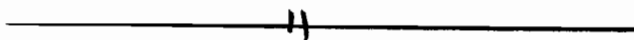
$$\Rightarrow \frac{du}{dx} = \cos x$$

$$\Rightarrow du = \cos x \, dx$$

$$= \int 8 u^4 \, du$$

$$= \frac{8u^5}{5} + C$$

$$= \frac{8}{5} \sin^5 x + C$$



INTEGRATION BY SUBSTITUTION

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6. 
$$\int_5^{10} \frac{x}{\sqrt{x-1}} dx$$

Let  $u = x - 1$

$$\Rightarrow \frac{du}{dx} = 1$$

$$\Rightarrow du = dx$$

Also  $x = u + 1$

Change limits

When  $x = 10$ ,  $u = 9$   
when  $x = 5$ ,  $u = 4$

$$= \int_4^9 \frac{u+1}{u^{1/2}} du$$

$$= \int_4^9 (u^{1/2} + u^{-1/2}) du$$

$$= \left[ \frac{u^{3/2}}{3/2} + \frac{u^{1/2}}{1/2} \right]_4^9$$

$$= \left[ \frac{2}{3} u^{3/2} + 2u^{1/2} \right]_4^9$$

$$= \left( \frac{2}{3} (9)^{3/2} + 2(9)^{1/2} \right) - \left( \frac{2}{3} (4)^{3/2} + 2(4)^{1/2} \right)$$

$$= \left( \frac{2}{3} (27) + 2(3) \right) - \left( \frac{2}{3} (8) + 2(2) \right)$$

$$= (18 + 6) - \left( \frac{16}{3} + 4 \right)$$

$$= 24 - \frac{16}{3} - 4$$

$$= 14\frac{2}{3}$$

INTEGRATION BY SUBSTITUTION

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7.  $\int_0^1 5x^2 e^{x^3} dx$

Let  $u = x^3$

$\Rightarrow \frac{du}{dx} = 3x^2$

$\Rightarrow du = 3x^2 dx$

$\Rightarrow \frac{1}{3} du = x^2 dx$

Change limits

When  $x = 1$ ,  $u = 1$

When  $x = 0$ ,  $u = 0$

$= \int_0^1 5 e^u \frac{1}{3} du$

$= \left[ \frac{5}{3} e^u \right]_0^1$

$= \frac{5}{3} e^1 - \frac{5}{3} e^0$

$= \frac{5}{3} e - \frac{5}{3}$

$= \frac{5}{3} (e - 1)$

