INTEGRATION BY SUBSTITUTION

Exercise

1. $\int x(2x-1)^3\,dx$

2. $\int \frac{2x}{x+3}\,dx$

3. $\int 2x\sqrt{4x+1}\,dx$

4. $\int 2x\sqrt{3x^2+1}\,dx$

5. $\int 8\cos x \sin^4 x\,dx$

6. $\int_5^{10} \frac{x}{\sqrt{x-1}}\,dx$

7. $\int_0^1 5x^2 e^{x^3}\,dx$

INTEGRATION BY SUBSTITUTION

1. $\int x(2x-1)^3\,dx$

Let $u = 2x-1$

$\Rightarrow \frac{du}{dx} = 2$

$\Rightarrow du = 2dx$

$\Rightarrow \frac{1}{2}du = dx$

Also $u+1 = 2x$

$\Rightarrow \frac{u+1}{2} = x$

$= \int \left(\frac{u+1}{2}\right) u^3 \frac{1}{2}\,du$

$= \frac{1}{4}\int (u+1)u^3\,du$

$= \frac{1}{4}\int (u^4+u^3)\,du$

$= \frac{1}{4}\left(\frac{u^5}{5}+\frac{u^4}{4}\right) + c$

$= \frac{u^5}{20} + \frac{u^4}{16} + c$

$= \frac{(2x-1)^5}{20} + \frac{(2x-1)^4}{16} + c$

2. $\int \frac{2x}{x+3}\,dx$

Let $u = x+3$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Also $x = u-3$

$= \int \frac{2(u-3)}{u}\,du$

$= \int \frac{2u-6}{u}\,du$

$= \int \left(\frac{2u}{u} - \frac{6}{u}\right) du$

$= \int \left(2 - \frac{6}{u}\right) du$

$= 2u - 6\ln u + c$

$= 2(x+3) - 6\ln|x+3| + c$

$= 2x + 6 - 6\ln|x+3| + c$

$= 2x - 6\ln|x+3| + c$ since constant c can absorb constant $+6$

3. $\displaystyle\int 2x\sqrt{4x+1}\,dx$

Let $u = 4x+1$

$\Rightarrow \dfrac{du}{dx} = 4$

$\Rightarrow du = 4\,dx$

$\Rightarrow \frac{1}{2}du = 2dx$

Also $u-1 = 4x$

$\Rightarrow \dfrac{u-1}{4} = x$

$$= \int \left(\frac{u-1}{4}\right) u^{\frac{1}{2}} \frac{1}{2}\,du$$

$$= \frac{1}{8}\int (u-1)\,u^{\frac{1}{2}}\,du$$

$$= \frac{1}{8}\int \left(u^{3/2} - u^{1/2}\right) du$$

$$= \frac{1}{8}\left(\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right) + c$$

$$= \frac{1}{8}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + c$$

$$= \frac{1}{20}u^{5/2} - \frac{1}{12}u^{3/2} + c$$

$$= \frac{1}{20}(4x+1)^{5/2} - \frac{1}{12}(4x+1)^{3/2} + c$$

INTEGRATION BY SUBSTITUTION

4. $\int 2x\sqrt{3x^2+1}\,dx$

Let $u = 3x^2+1$

$\Rightarrow \frac{du}{dx} = 6x$

$\Rightarrow du = 6x\,dx$

$\Rightarrow \frac{1}{3}du = 2x\,dx$

$= \int u^{\frac{1}{2}} \frac{1}{3}\,du$

$= \frac{1}{3}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{3}\left(\frac{u^{3/2}}{\frac{3}{2}}\right) + c$

$= \frac{1}{3}\left(\frac{2}{3}u^{3/2}\right) + c$

$= \frac{2}{9}u^{3/2} + c$

$= \frac{2}{9}\left(3x^2+1\right)^{3/2} + c$

5. $\int 8\cos x \sin^4 x \, dx$

Let $u = \sin x$

$\Rightarrow \frac{du}{dx} = \cos x$

$\Rightarrow du = \cos x \, dx$

$= \int 8u^4 \, du$

$= \frac{8u^5}{5} + C$

$= \frac{8}{5}\sin^5 x + C$

6. $\displaystyle\int_5^{10} \frac{x}{\sqrt{x-1}}\,dx$

Let $u = x-1$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Also $x = u+1$

Change limits

When $x=10$, $u=9$
when $x=5$, $u=4$

$$= \int_4^9 \frac{u+1}{u^{1/2}}\,du$$

$$= \int_4^9 \left(u^{\frac{1}{2}} + u^{-\frac{1}{2}}\right)du$$

$$= \left[\frac{u^{3/2}}{3/2} + \frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^9$$

$$= \left[\frac{2}{3}u^{3/2} + 2u^{\frac{1}{2}}\right]_4^9$$

$$= \left(\frac{2}{3}(9)^{3/2} + 2(9)^{\frac{1}{2}}\right) - \left(\frac{2}{3}(4)^{3/2} + 2(4)^{\frac{1}{2}}\right)$$

$$= \left(\frac{2}{3}(27) + 2(3)\right) - \left(\frac{2}{3}(8) + 2(2)\right)$$

$$= (18 + 6) - \left(\frac{16}{3} + 4\right)$$

$$= 24 - \frac{16}{3} - 4$$

$$= 14\tfrac{2}{3}$$

7. $\int_0^1 5x^2 e^{x^3} dx$

Let $u = x^3$

$\Rightarrow \frac{du}{dx} = 3x^2$

$\Rightarrow du = 3x^2 dx$

$\Rightarrow \frac{1}{3} du = x^2 dx$

Change limits

When $x = 1$, $u = 1$
When $x = 0$, $u = 0$

$= \int_0^1 5 e^u \frac{1}{3} du$

$= \left[\frac{5}{3} e^u \right]_0^1$

$= \frac{5}{3} e^1 - \frac{5}{3} e^0$

$= \frac{5}{3} e - \frac{5}{3}$

$= \frac{5}{3}(e - 1)$