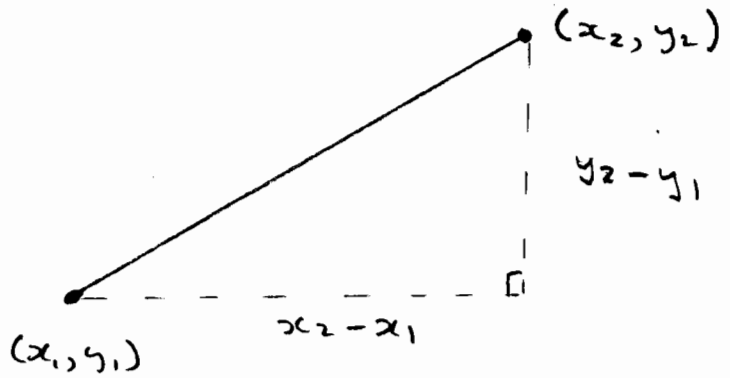
COORDINATE GEOMETRY BASICS

1. Gradient of straight line joining (x_1, y_1) and (x_2, y_2)



$$\text{gradient } m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1 Find gradient of line joining $(2, 5)$ and $(7, 13)$

Let $(x_1, y_1) = (2, 5)$ and $(x_2, y_2) = (7, 13)$

Then $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{13 - 5}{7 - 2} = \frac{8}{5}$$

Example 2 Find gradient of line joining $(-3, 4)$ and $(7, -5)$

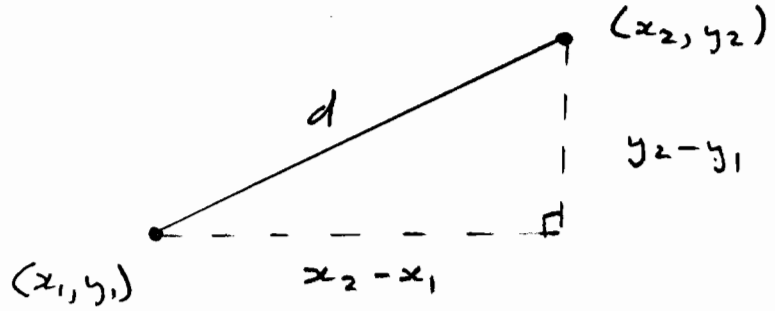
Let $(x_1, y_1) = (-3, 4)$ and $(x_2, y_2) = (7, -5)$

Then $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 4}{7 - -3} = \frac{-9}{10}$$

$$\therefore m = -\frac{9}{10}$$

COORDINATE GEOMETRY BASICS

2. Distance between two points (x_1, y_1) and (x_2, y_2)



Let distance be d

Then by Pythagoras

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1

Find distance between $(2, 5)$ and $(7, 13)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(7-2)^2 + (13-5)^2}$$

$$d = \sqrt{5^2 + 8^2} = \sqrt{25+64} = \sqrt{89}$$

Example 2

Find distance between $(-3, 4)$ and $(7, -5)$

$$d = \sqrt{(7--3)^2 + (-5-4)^2}$$

$$d = \sqrt{10^2 + (-9)^2} = \sqrt{100+81} = \sqrt{181}$$

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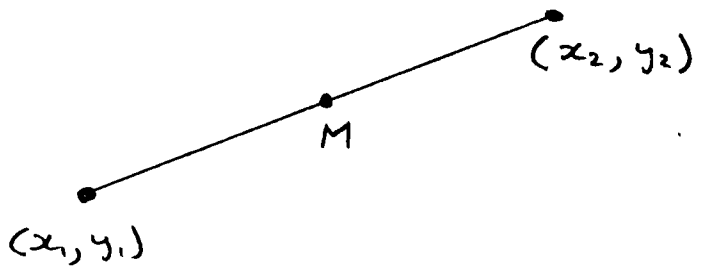
3. Midpoint between two points (x_1, y_1) and (x_2, y_2)



The midpoint has coordinates which are the average of the corresponding coordinates of each end point

$$M = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$$

Example 1

Find midpoint of $(2,5)$ and $(7,13)$

$$M = \left(\frac{2+7}{2} , \frac{5+13}{2} \right)$$

$$M = \left(\frac{9}{2} , 9 \right)$$

Example 2

Find midpoint of $(-3,4)$ and $(7,-5)$

$$M = \left(\frac{-3+7}{2} , \frac{4+(-5)}{2} \right)$$

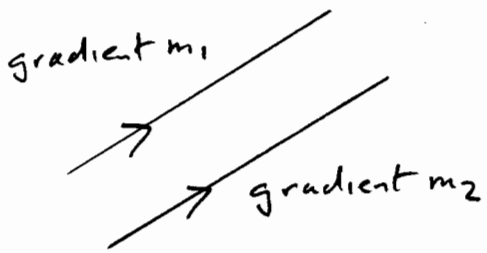
$$M = \left(\frac{-3+7}{2} , \frac{4-5}{2} \right)$$

$$M = \left(2 , -\frac{1}{2} \right)$$

COORDINATE GEOMETRY BASICS

4.

Parallel lines have the same gradient

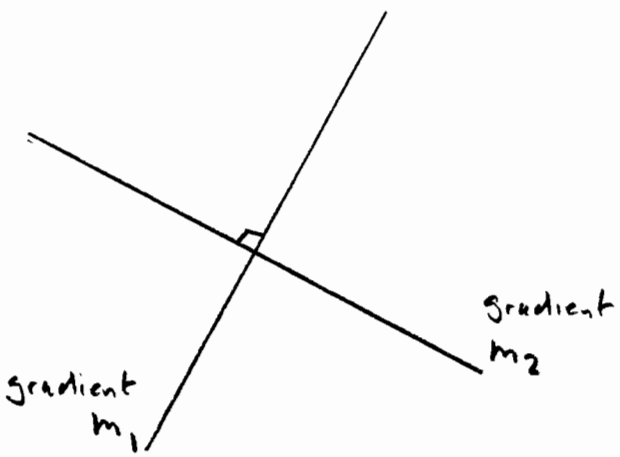


If parallel then $m_1 = m_2$

Examples of parallel lines

$$\begin{cases} y = 2x + 3 \\ y = 2x - 7 \end{cases} \qquad \begin{cases} y = -\frac{1}{3}x - 4 \\ y = -\frac{1}{3}x + 1 \end{cases}$$

Perpendicular lines have gradients that multiply to -1



$$m_1 \times m_2 = -1$$

$$\text{or} \quad m_2 = \frac{-1}{m_1}$$

Examples of perpendicular lines

$$\begin{cases} y = 3x + 4 \\ y = -\frac{1}{3}x - 7 \end{cases} \qquad \begin{cases} y = \frac{5}{4}x + 1 \\ y = -\frac{4}{5}x - 3 \end{cases}$$

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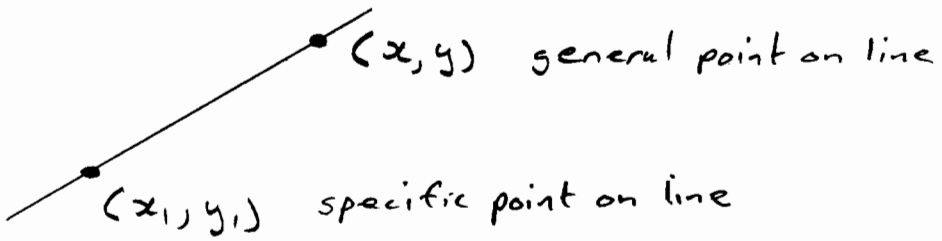
5. Equation of line with gradient m passing through point (x_1, y_1)



Gradient $m = \dfrac{y - y_1}{x - x_1}$

$\therefore \quad y - y_1 = m(x - x_1)$

Example 1 Find eqn of line with gradient 5 thro point (7,4)

Using $y - y_1 = m(x - x_1)$

$y - 4 = 5(x - 7)$

$y - 4 = 5x - 35$

$y = 5x - 35 + 4$

$y = 5x - 31$

Example 2 Find eqn of line with gradient $-\frac{1}{2}$ thro (4,6)

Using $y - y_1 = m(x - x_1)$

$y - 6 = -\frac{1}{2}(x - 4)$

$y - 6 = -\frac{1}{2}x + 2$

$y = -\frac{1}{2}x + 2 + 6$

$y = -\frac{1}{2}x + 8$

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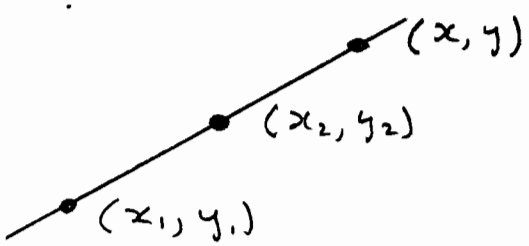
6. Equation of line through given points (x_1, y_1) and (x_2, y_2)



$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{but also} \qquad m = \frac{y - y_1}{x - x_1}$$

$$\therefore \quad \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$\Rightarrow \quad \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Example 1 Find equation of line through $(3, 9)$ and $(5, 13)$

Using $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

$$\frac{y - 9}{13 - 9} = \frac{x - 3}{5 - 3}$$

$$\Rightarrow \quad \frac{y - 9}{4} = \frac{x - 3}{2}$$

$$\Rightarrow \quad y - 9 = \frac{4(x - 3)}{2}$$

$$\Rightarrow \quad y - 9 = 2(x - 3)$$

$$\Rightarrow \quad y - 9 = 2x - 6$$

$$\Rightarrow \quad y - 9 = 2x - 6 + 9$$

$$\Rightarrow \quad y = 2x + 3$$
