GEOMETRIC PROGRESSIONS

TRANSCRIPT

Example

$$6,\ 12,\ 24,\ 48,\ 96,\ \ldots\ldots$$

Characterised by:

1st term: 6

Common ratio: 2

In general:

1st term	a
2nd term	ar
3rd term	ar^2
4th term	ar^3
....	
n^{th} term	ar^{n-1}

Finding the sum of the first n terms:

Consider

$$S_n = a + \cancel{ar} + \cancel{ar^2} + \cancel{ar^3} + \cancel{\ldots} + \cancel{ar^{n-2}} + \cancel{ar^{n-1}} \qquad (1)$$

then

$$rS_n = \cancel{ar} + \cancel{ar^2} + \cancel{ar^3} + \cancel{ar^4} + \cancel{\ldots} + \cancel{ar^{n-1}} + ar^n \qquad (2)$$

If we subtract (2) − (1) all except 2 terms on right cancel

$$rS_n - S_n = ar^n - a$$

$$(r-1)S_n = a(r^n - 1)$$

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{a(1-r^n)}{1-r}$$

We tend to use $S_n = \dfrac{a(r^n-1)}{r-1}$ when $|r|>1$

and $S_n = \dfrac{a(1-r^n)}{1-r}$ when $|r|<1$.

Sum to Infinity

If $|r|<1$ then $r^n \to 0$ as $n \to \infty$

$$\Rightarrow \quad S_\infty = \frac{a(1-0)}{1-r} = \frac{a}{1-r}$$

The sum to infinity exists only when $|r|<1$

then $S_\infty = \dfrac{a}{1-r}$

Example 1

For the GP: 5, 15, 45, 135, ...

Find the 8^{th} term and the sum of the first 6 terms, S_6

$a = 5, \quad r = 3$

8^{th} term $= ar^7$

$= 5 \times 3^7$

$= 10{,}935$

$S_n = \dfrac{a(r^n-1)}{r-1}$

$S_6 = \dfrac{5(3^6-1)}{3-1}$

$S_6 = 1{,}820$

Example 2

For the GP: 40, 8, 1.6, 0.32, ...

Find the sum to infinity, S_∞

$$a = 40, \quad r = \frac{8}{40} = \frac{1}{5}$$

$$S_\infty = \frac{a}{1-r} = \frac{40}{1-\frac{1}{5}} = \frac{40}{\frac{4}{5}} = 40 \times \frac{5}{4} = 50$$

$$S_\infty = 50$$

Example 3

The 3rd term of a GP is 18 and the 8^{th} term is 4374. Find the sum of the first 10 terms, S_{10}.

8^{th} term $= ar^7 = 4374$

3^{rd} term $= ar^2 = 18$

$$\Rightarrow \frac{ar^7}{ar^2} = \frac{4374}{18}$$

$$\Rightarrow r^5 = 243$$

$$\Rightarrow r = 243^{1/5}$$

$$\Rightarrow \underline{r = 3}$$

Sub in 3rd term

$$a \times 3^2 = 18$$

$$9a = 18$$

$$\underline{a = 2}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{2(3^{10} - 1)}{3 - 1}$$

$$S_{10} = 59,048$$

Example 4

The 1^{st} term of a GP is 60 and the sum to infinity is 100. Find the common ratio.

$a = 60$

$$S_\infty = \frac{a}{1-r} = 100$$

$$\therefore \quad \frac{60}{1-r} = 100$$

$$\Rightarrow \quad 60 = 100(1-r)$$

$$\Rightarrow \quad 60 = 100 - 100r$$

$$\Rightarrow \quad 100r = 100 - 60$$

$$\Rightarrow \quad 100r = 40$$

$$\Rightarrow \quad r = \frac{40}{100}$$

$$\Rightarrow \quad r = 0.4$$

Example 5

The 1^{st} term of a GP is 4 and the common ratio is 7. Which term is the first to exceed 100,000?

$a = 4, \quad r = 7$

Let n^{th} term be first to exceed 100,000

Then $\quad ar^{n-1} > 100{,}000$

$$4 \times 7^{n-1} > 100{,}000$$

$$7^{n-1} > 25{,}000$$

Method 1 - trial and error

$$7^{10} = 282,475,249$$

$$7^5 = 16,807 \quad < 25000$$

$$7^6 = 117,649 \quad > 25000$$

$$\therefore \quad n-1 = 6$$

$$\Rightarrow n = 7$$

7^{th} term first to exceed 100,000

Method 2 - Using logarithms

$$7^{n-1} > 25000$$

$$\log_{10} 7^{n-1} > \log_{10} 25000$$

$$(n-1)\log_{10} 7 > \log_{10} 25000$$

$$n-1 > \frac{\log_{10} 25000}{\log_{10} 7}$$

$$n-1 > 5.2$$

$$\Rightarrow \quad n > 6.2$$

$$\Rightarrow \quad n = 7$$

Example 6

In the GP: 3, 6, 12, 24, ...

how many terms are required for the sum to exceed 1,000,000?

$a = 3$, $r = 2$ Let number of terms be n

$$\Rightarrow S_n > 1{,}000{,}000 \quad \Rightarrow \frac{a(r^n - 1)}{r-1} > 1{,}000{,}000$$

$$\Rightarrow \frac{3(2^n - 1)}{2-1} > 1{,}000{,}000$$

$$\Rightarrow 2^n - 1 > \frac{1{,}000{,}000}{3}$$

$$\Rightarrow 2^n > 333{,}333.3 + 1$$

$$\Rightarrow 2^n > 333{,}334.3$$

Method 1 - trial and error

$$2^{12} = 4096$$

$$2^{16} = 65{,}536$$

$$2^{18} = 262{,}144$$

$$2^{19} = 524{,}288$$

$$\Rightarrow 2^{19} > 333{,}334.3$$

so 19 terms required

Method 2 - using logarithms

$$2^n > 333{,}334.3$$

$$\Rightarrow \log 2^n > \log 333{,}334.3$$

$$\Rightarrow n \log 2 > \log 333{,}334.3$$

$$\Rightarrow n > \frac{\log 333{,}334.3}{\log 2}$$

$$\Rightarrow n > 18.3$$

$$\Rightarrow n = 19$$

so 19 terms required