INTEGRATION BY PARTS

Ex 1

$$\int 3x \sin 4x \, dx \qquad \text{Using } \int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$$

Let $u = 3x$ $\qquad$ Let $\frac{dv}{dx} = \sin 4x$

$\Rightarrow \frac{du}{dx} = 3$ $\qquad$ $\Rightarrow v = -\frac{1}{4}\cos 4x$

$$\int 3x \sin 4x \, dx = -\frac{3}{4}\cos 4x + \int \frac{3}{4}\cos 4x \, dx$$

$$= -\frac{3}{4}\cos 4x + \frac{3}{16}\sin 4x + c$$

INTEGRATION BY PARTS

Ex 2

$$\int (2x-3)e^{5x}\,dx \qquad \text{Using } \int v\frac{du}{dx} = uv - \int v\frac{du}{dx}$$

Let $u = 2x - 3$ $\qquad$ Let $\frac{dv}{dx} = e^{5x}$

$\Rightarrow \frac{du}{dx} = 2$ $\qquad$ $\Rightarrow v = \frac{1}{5}e^{5x}$

$$\int (2x-3)e^{5x}dx = (2x-3)\frac{1}{5}e^{5x} - \int \frac{2}{5}e^{5x}dx$$

$$= \left(\frac{2x-3}{5}\right)e^{5x} - \frac{2}{25}e^{5x} + c$$

$$= \left(\frac{10x-15}{25}\right)e^{5x} - \frac{2}{25}e^{5x} + c$$

$$= \left(\frac{10x-17}{25}\right)e^{5x} + c$$

INTEGRATION BY PARTS

Ex 3

$\int x^3 \ln x \, dx$ using $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

Let $u = \ln x$ Let $\frac{dv}{dx} = x^3$

$\Rightarrow \frac{du}{dx} = \frac{1}{x}$ $\Rightarrow v = \frac{x^4}{4}$

$$\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \int \frac{x^4}{4} \cdot \frac{1}{x} \, dx$$

$$= \frac{x^4}{4} \ln x - \int \frac{x^3}{4} \, dx$$

$$= \frac{x^4}{4} \ln x - \frac{x^4}{16} + c$$

INTEGRATION BY PARTS

Ex 4

$$\int \frac{1}{x^3} \ln(5x)\,dx \qquad \text{using} \quad \int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$$

Let $u = \ln(5x)$

$\Rightarrow \frac{du}{dx} = \frac{5}{5x} = \frac{1}{x}$

Let $\frac{dv}{dx} = \frac{1}{x^3} = x^{-3}$

$\Rightarrow v = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$

$$\int \frac{1}{x^3} \ln(5x)\,dx = -\frac{1}{2x^2}\ln(5x) + \int \frac{1}{2x^2}\cdot\frac{1}{x}\,dx$$

$$= -\frac{1}{2x^2}\ln(5x) + \int \frac{1}{2x^3}\,dx$$

$$= -\frac{1}{2x^2}\ln(5x) + \int \frac{x^{-3}}{2}\,dx$$

$$= -\frac{1}{2x^2}\ln(5x) + \frac{x^{-2}}{2(-2)} + c$$

$$= -\frac{1}{2x^2}\ln(5x) - \frac{1}{4x^2} + c$$

//

INTEGRATION BY PARTS

Ex 5

$$\int_0^{\frac{\pi}{3}} x\cos x\,dx \qquad \text{using} \quad \int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$$

Let $u = x$ $\qquad$ Let $\frac{dv}{dx} = \cos x$

$\Rightarrow \frac{du}{dx} = 1$ $\qquad$ $\Rightarrow v = \sin x$

$$\int_0^{\frac{\pi}{3}} x\cos x\,dx = \Big[x\sin x\Big]_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} \sin x\,dx$$

$$= \Big[x\sin x\Big]_0^{\frac{\pi}{3}} - \Big[-\cos x\Big]_0^{\frac{\pi}{3}}$$

$$= \left[\frac{\pi}{3}\sin\frac{\pi}{3} - 0\sin 0\right] - \left[-\cos\frac{\pi}{3} - (-\cos 0)\right]$$

$$= \frac{\pi}{3}\times\frac{\sqrt{3}}{2} - 0 + \frac{1}{2} - 1$$

$$= \frac{\sqrt{3}\,\pi}{6} - \frac{1}{2}$$

INTEGRATION BY PARTS

Ex 6 $\displaystyle\int_0^1 2x\,e^{x+1}\,dx$ using $\displaystyle\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$

Let $u = 2x$ Let $\dfrac{dv}{dx} = e^{x+1}$

$\Rightarrow \dfrac{du}{dx} = 2$ $\Rightarrow v = e^{x+1}$

$$\int_0^1 2x\,e^{x+1}\,dx = \Big[2x\,e^{x+1}\Big]_0^1 - \int_0^1 2\,e^{x+1}\,dx$$

$$= \Big[2x\,e^{x+1}\Big]_0^1 - \Big[2e^{x+1}\Big]_0^1$$

$$= \Big[2e^2 - 0\Big] - \Big[2e^2 - 2e\Big]$$

$$= 2e^2 - 2e^2 + 2e$$

$$= 2e$$

INTEGRATION BY PARTS

Ex 7 $\displaystyle\int_1^2 4x \ln x \, dx$ using $\displaystyle\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$

Let $u = \ln x$ Let $\dfrac{dv}{dx} = 4x$

$\Rightarrow \dfrac{du}{dx} = \dfrac{1}{x}$ $\Rightarrow v = 2x^2$

$$\int_1^2 4x \ln x \, dx = \left[2x^2 \ln x\right]_1^2 - \int_1^2 2x^2 . \frac{1}{x} \, dx$$

$$= \left[2x^2 \ln x\right]_1^2 - \int_1^2 2x \, dx$$

$$= \left[2x^2 \ln x\right]_1^2 - \left[x^2\right]_1^2$$

$$= \left[8\ln 2 - 2\ln 1\right] - \left[4 - 1\right]$$

$$= 8\ln 2 - 0 - 4 + 1$$

$$= 8\ln 2 - 3$$